The Birthday Paradox

r	$P_{match}(r)$
1	0.000
2	0.003
3	0.008
4	0.016
5	0.027
6	0.040
7	0.056
8	0.074
9	0.095
10	0.117
11	0.141
12	0.167
13	0.194
14	0.223
15	0.253
16	0.284
17	0.315
18	0.347
19	0.379
20	0.411
21	0.444
22	0.476
23	0.507
24	0.538
25	0.569
26	0.598
27	0.627
28	0.654
29	0.681
30	0.706

The probability that two strangers do not share a birthday is $\frac{364}{365}$, assuming that neither of them was born in a leap year.

The probability of a match is the complement of this event, that is, $1-\frac{364}{365}$, or .00274.

The probability that three strangers do not share a birthday is $\frac{364}{365}\times\frac{363}{365}$, or .991796, with the complement, where a pair of matching birthdays exists, being $1-(\frac{364}{365}\times\frac{363}{365}) = 0.008204$.

Considering a group of four strangers where a matching pair is present produces the probability
$1-(\frac{364}{365}\times\frac{363}{365}\times\frac{362}{365})= 0.016356$

At this stage, the students should generate a general formula that calculates the probability of a matching pair of birthdays for *r* random birthdays. The general formula involves permutations and is:

$$P_{match}(r) = 1 - \frac{{}_{365}P_r}{365^r},$$

where *r* is the number of random birthdays.

This result can be calculated with handheld calculators for the first few values of *r*, but larger values of *r* may be beyond the capabilities of most calculators. The table feature of the TI-83, for example, handles the calculation through *r* = 39, but at *r* = 40 the calculator fails because of overflow problems. At this point, students could turn to the computer and generate a spreadsheet, which can handle the large computations. The spreadsheet can then be used to calculate the probabilities and create a scatter-plot of the results (Lesser 1999).